Short Papers

A Method for Evaluating and the Evaluation of the Influence of the Reverse Transfer Gain on the Transducer Power Gain of Some Microwave Transistors

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Abstract—**The unilateral figure of merit is frequently used to decide whether, from the viewpoint of transducer power gain, a transistor can be considered to be unilateral or not. In this communication, the limitations of this measure are discussed and alternative measures, as well as a procedure for determining the exact bounds on the gain error $G_T/G_{T,u}$, are proposed and applied to several microwave transistors in order to evaluate the influence of the reverse transfer gain on the transducer power gain.**

I. Introduction

The unilateral figure of merit [1]–[3]

$$u=\left|\frac{s_{11}s_{12}s_{21}s_{22}}{\left[1-|s_{11}|^2\right]\left[1-|s_{22}|^2\right]}\right| \tag{1}$$

is frequently used to decide whether, from the viewpoint of transducer power gain, a transistor can be considered to be unilateral or not. The ratio of the actual transducer power gain (G_T) to that of the gain under the assumption that the transistor is unilateral ($G_{T,u}$) is then taken to be [1]–[3]

$$\frac{1}{[1+u]^2}\leqslant\frac{G_T}{G_{T,u}}\leqslant\frac{1}{[1-u]^2}. \tag{2}$$

It will be shown here that the bounds as given by this equation are optimistic and do not provide an adequate measure of the influence of the internal feedback in a transistor, especially when both the load and the source are mismatched to the transistor as is often the case in multistage amplifiers. Alternative measures will be proposed, and a procedure for determining the exact bounds on the gain error will be outlined and applied to several microwave transistors in order to evaluate the influence of the reverse transfer gain on the transducer power gain.

II. Limitations of the Unilateral Figure of Merit

The bounds given by (1) and (2) were derived from the equation [1]–[3]

$$\frac{G_T}{G_{T,u}}=\frac{|1-\Gamma_s s_{11}|^2|1-\Gamma_L s_{22}|^2}{|(1-\Gamma_s s_{11})(1-\Gamma_L s_{22})-\Gamma_s\Gamma_L s_{11}s_{22}|^2}. \tag{3}$$

It is a simple matter to show that

$$\frac{G_T}{G_{T,u}}=\frac{1}{|1-U(\Gamma_s,\Gamma_L)|^2} \tag{4}$$

where

$$U(\Gamma_s,\Gamma_L)=\frac{\Gamma_s\Gamma_L s_{12}s_{21}}{(1-\Gamma_s s_{11})(1-\Gamma_L s_{22})}. \tag{5}$$

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In order to derive (2), the inequality

$$\frac{1}{(1+|U|)^2}\leqslant\frac{1}{|1-U|^2}\leqslant\frac{1}{(1-|U|)^2} \tag{6}$$

was used, and U was taken to be

$$u=|U(s_{11}^*,s_{22}^*)|. \tag{7}$$

If the magnitude of $U(\Gamma_s,\Gamma_L)$ was always smaller than u and $|u|<1$, (2) would at least have provided an upper and a lower bound for the ratio of the true and the unilateralized gain, even though it would be inaccurate, since (6) is used to simplify the problem of determining the bounds.

By writing (5) in the form

$$U(\Gamma_s,\Gamma_L)=\frac{s_{12}s_{21}}{\left(\frac{1}{\Gamma_s}-s_{11}\right)\left(\frac{1}{\Gamma_L}-s_{22}\right)} \tag{8}$$

and taking

$$\Gamma_i=|a_i|s_{ii}^* \tag{9}$$

where

$$|a_i|>1$$

it can be shown easily that u is not the upper bound for the magnitude of $U(\Gamma_s,\Gamma_L)$.

Apart from being optimistic, the unilateral figure of merit has the further disadvantage that it does not distinguish between the various load and source conditions which can exist, that is, the load or the source or both can be mismatched and the degree of mismatch can take on specific values or, when a transistor is evaluated for all load and/or source conditions, arbitrary values.

By plotting the vector

$$d_s=\frac{1}{\Gamma_s}-s_{11} \tag{10}$$

on a Smith Chart, it becomes clear that the required upper bound on $|U(\Gamma_s,\Gamma_L)|$ for arbitrary Γ_s and Γ_L is indeed

$$u'=\left|U\left(\frac{s_{11}^*}{|s_{11}|},\frac{s_{22}^*}{|s_{22}|}\right)\right|=\left|\frac{s_{21}s_{12}}{(1-|s_{11}|)(1-|s_{22}|)}\right|. \tag{11}$$

When only the source or the load is to be mismatched, the upper bound on the magnitude of $U(\Gamma_s,\Gamma_L)$ is given approximately by

$$u_s'=\left|\frac{s_{12}s_{21}s_{22}}{\left[1-|s_{11}|\right]\left[1-|s_{22}|^2\right]}\right| \tag{12}$$

and

$$u_L'=\left|\frac{s_{11}s_{12}s_{21}}{\left[1-|s_{11}|^2\right]\left[1-|s_{22}|\right]}\right| \tag{13}$$

respectively. In deriving these equations, the termination at the matched port was taken to be $\Gamma_s=s_{11}^*$ or $\Gamma_L=s_{22}^*$, as applicable.

Equations (11)–(13) can be used in conjunction with (6) to find upper and lower bounds for the gain ratio $G_T/G_{T,u}$ under the different load and source conditions, that is, as long as the magnitude of the corresponding modified figure of merit is less

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than one, which will be the case if the transistor can be considered to be unilateral.

III. An Iterative Procedure for Determining the Exact Bounds on the Gain Ratio $G_T/G_{T,u}$

Although (11)–(13) used in conjunction with (6) provide upper and lower bounds for the gain ratio $G_T/G_{T,u}(|u'|, |u'_s|, |u'_L| < 1)$ and can be used as a screening test for unilateralness, the resulting bounds are pessimistic and it may turn out that a transistor which is apparently nonunilateral can in fact be considered to be unilateral. The basic reason for the pessimistic results is that resort was taken to (6) in order to arrive at simple measures.

An analytical procedure for determining the exact bounds on the gain ratio for a particular design value of the unilateral gain $G_{T,u}$ when only the load or the source impedance is varied and the termination at the other port is such that $\Gamma_s = s_{11}^*$ or $\Gamma_L = s_{22}^*$, was outlined in [4]. Considering the bounds for a particular design value of $G_{T,u}$ is a useful refinement, since the bounds can be a strong function of the gain. It is, however, also useful to have the exact bounds for an arbitrary load and/or source impedance. An iterative procedure for determining these bounds, as well as those for specified gains G_1 and/or G_2 will be derived here.

$U(\Gamma_s, \Gamma_L)$ as defined by (8) can be written in the form

$$U(\Gamma_s, \Gamma_L) = \frac{s_{12}s_{21}}{\left(\frac{1}{\Gamma_s} - s_{11}\right)\left(\frac{1}{\Gamma_L} - s_{22}\right)}$$

$$= \frac{s_{12}s_{21}}{[|b|(\cos\alpha + j\sin\alpha) - s_{11}][|c|(\cos\beta + j\sin\beta) - s_{22}]} \quad (14)$$

where

$$|b|, |c| \geq 1.$$

It can be shown easily that for any particular α and β, $|U(\Gamma_s, \Gamma_L)|$ will be a maximum when

$$|b| = 1 = |c| \quad (15)$$

that is, if

$$|s_{11}|, |s_{22}| < 1.$$

The maximum value of $U(\Gamma_s, \Gamma_L)$ corresponding to a particular α and β can therefore be determined by calculating

$$U(\alpha, \beta) = \frac{s_{12}s_{21}}{(\cos\alpha + j\sin\alpha - s_{11})(\cos\beta + j\sin\beta - s_{22})}. \quad (16)$$

Having calculated $U(\alpha, \beta)$, the corresponding value of $|1 - U(\alpha, \beta)|$ can be calculated. As long as the maximum value of $|1 - U(\alpha, \beta)|$ is bigger than one, the maximum value of $|1 - U(\alpha, \beta)|$ is also the maximum value of $|1 - U(\Gamma_s, \Gamma_L)|$.

In order to find the minimum value of $|1 - U(\Gamma_s, \Gamma_L)|$, it is necessary to find the minimum value of $|1 - U(\alpha, \beta)|$, and when

$$|U(\alpha, \beta)| > \left|\cos\tan^{-1}\left[\frac{\text{Imag} - U(\alpha, \beta)}{\text{Real} - U(\alpha, \beta)}\right]\right| \quad (17)$$

and

$$\text{Real}[-U(\alpha, \beta)] < 0$$

the minimum value of $|1 - U(\alpha, \beta)|$ and the distance

$$D_m = \left|\sin\left[\tan^{-1}\frac{\text{Imag} - U(\alpha, \beta)}{\text{Real} - U(\alpha, \beta)}\right]\right|. \quad (18)$$

The reason for (18) is illustrated in Fig. 1: When the situation depicted occurs, there will always be a Γ_{sy} and Γ_{Ly} for which

$$D_m = |1 - U(\Gamma_{sy}, \Gamma_{Ly})| < |1 - U(\alpha, \beta)|.$$

By using (16)–(18), it is a simple matter to develop a computer program which can be used to determine the minimum and maximum value of $|1 - U(\Gamma_s, \Gamma_L)|$ and therefore the maximum and minimum value of $G_T/G_{T,u}$ to good approximation for arbitrary Γ_L and Γ_s.

The bounds, when either the source or the load reflection coefficient is known and the other can take on an arbitrary value, can be determined by varying only α and β as appropriate and setting the other reflection coefficient equal to the specified value.

When the bounds with G_1 and/or G_2 specified are required, $|1 - U(\Gamma_s, \Gamma_L)|$ must be evaluated for

$$0 \leq \theta_i < 2\pi, \qquad i = 1 \text{ or } 2 \text{ or } 1,2$$

where

$$\Gamma_i = \frac{g_i|s_{ii}|}{1 - |s_{ii}|^2(1 - g_i)} \frac{s_{ii}^*}{|s_{ii}|} + \frac{\sqrt{1 - g_i}\,(1 - |s_{ii}|^2)}{1 - |s_{ii}|^2(1 - g_i)} e^{j\theta_i} \quad (19)$$

$$g_i = G_i[1 - |s_{ii}|^2] \quad (20)$$

$$G_{T,u} = G_1|s_{21}|^2G_2 \quad (21)$$

$$\Gamma_s = \Gamma_1$$

and

$$\Gamma_L = \Gamma_2.$$

When the reflection coefficient Γ_j has a fixed value, G_j can be calculated by using

$$G_j = \frac{1 - |\Gamma_j|^2}{|1 - \Gamma_j s_{jj}|^2}. \quad (22)$$

Example 1

The exact bounds for the NE38806 GaAs FET at 4 GHz are compared below to the bounds resulting from (1), (11)–(13). It is clear from these that the bounds resulting from using the unilateral figure of merit are optimistic and those resulting from using (11)–(13) pessimistic. As a screening test, a pessimistic measure is, however, preferable.

Since $|u'| > 1$, the upper bound corresponding to it is meaningless.

Bounds resulting from u :	0.614–1.908
Bounds resulting from u'_s :	0.376–7.300
Exact bounds :	0.496–1.948
Bounds resulting from u'_L :	0.316–20.27
Exact bounds :	0.619–8.596
Bounds resulting from u' :	0.130–
Exact bounds :	0.164–∞.

It is clear from the exact bounds that the influence of the internal feedback of the NE38806 on the transducer power gain is not negligible when the load and/or source terminations can take on arbitrary values. Furthermore, mismatching of the load is more severe than mismatching of the source.

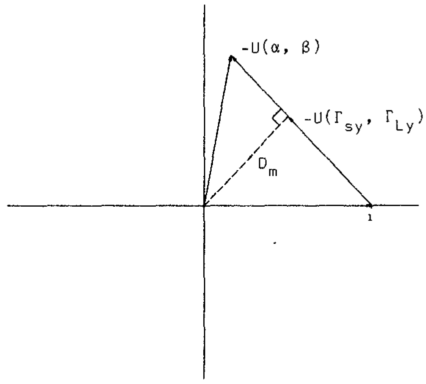

Fig. 1. The vector $|1-U(\alpha,\beta)|$ and the distance D_m: $D_m < |1-U(\alpha,\beta)|$.

TABLE I
BOUNDS ON THE GAIN RATIO $G_T/G_{T,u}$ OF THE NE38806 FOR DIFFERENT VALUES OF g_1 AND g_2

g_1, g_2	Lower and Upper Bound Corresponding to g_1	Lower and Upper Bound Corresponding to g_2
1.0	0.874 - 0.874	1.550 - 1.550
0.9	0.717 - 1.091	1.105 - 2.329
0.8	0.664 - 1.204	0.975 - 2.833
0.6	0.599 - 1.398	0.827 - 3.884
0.4	0.554 - 1.579	0.736 - 5.093
0.2	0.521 - 1.757	0.670 - 6.755

Example 2

The bounds for the gain ratio of the NE38806 for different values of g_1 ($\Gamma_L = s_{22}^*$) and g_2 ($\Gamma_s = s_{11}^*$) are summarized in Table I. It is interesting to note how the bounds increase as the gain decreases.

It is clear from the contents of Table I that the NE38806 cannot be considered to be unilateral at all.

IV. The Influence of the Reverse Transfer Gain on the Transducer Power Gain of Some Microwave Transistors

The exact bounds corresponding to arbitrary source and/or load terminations and $g_1 = 0.7943$ and/or $g_2 = 0.7943$ with $\Gamma_s = s_{11}^*$ or $\Gamma_L = s_{22}^*$ as applicable were determined for several microwave transistors and are tabulated in Table II. It appears from these results that very few microwave transistors can be considered to be unilateral.

TABLE II
THE EXACT BOUNDS ON THE GAIN RATIO $G_T/G_{T,u}$ FOR SEVERAL MICROWAVE TRANSISTORS UNDER DIFFERENT SOURCE AND LOAD CONDITIONS

Transistor	Γ_s $\Gamma_L = s_{22}^*$	Γ_L $\Gamma_s = s_{11}^*$	Γ_s Γ_L	$g_1=0.794$ $\Gamma_L = s_{22}^*$	$g_2=0.794$ $\Gamma_s = s_{11}^*$	$g_1=0.794$ $g_2=0.794$
MG2124 11.7GHz [5]	0.942 1.342	0.885 1.451	0.704 2.774	1.032 1.212	1.001 1.253	0.944 1.469
GAT1 GaAs FET 5V 10mA 2GHz	0.948 1.607	0.961 1.571	0.793 3.347	1.083 1.382	1.091 1.374	1.023 1.696
HXTR Bipolar Transistor 4GHz [2]	0.839 1.795	0.905 1.618	0.595 7.457	1.006 1.419	1.045 1.358	0.921 1.881
GaAs HFET-1000 12GHz [2]	0.824 2.056	0.845 1.977	0.522 11.88	1.018 1.533	1.031 1.510	0.869 2.167
NE70083 GaAs MESFET 3V 30mA 18GHz	0.863 1.662	0.763 2.000	0.471 17.02	1.012 1.359	0.951 1.463	0.834 2.070
GAT4 GaAs FET 5V 10mA 8GHz	0.612 1.787	0.544 2.212	0.253 59.61	0.777 1.255	0.732 1.356	0.549 2.029
NE71083 GaAs MESFET 3V 30mA 18GHz	0.875 1.862	0.764 2.316	0.482 94.36	1.048 1.473	0.979 1.602	0.882 2.481
HP 1u chip 10GHz [4]	0.448 1.507	0.458 1.441	0.213 101.2	0.579 1.169	0.589 1.188	0.427 1.610
NE720 GaAs MESFET 3V 10mA 4GHz	0.642 2.915	0.502 2.714	0.200 ∞	0.881 1.721	0.770 2.112	0.548 5.145
NE38806 GaAs FET 4GHz [2]	0.496 1.948	0.405 3.101	0.164 ∞	0.661 1.210	0.595 1.411	0.415 2.330
Plessey CCD device 8GHz [4]	0.682 18.35	0.884 5.540	0.348 ∞	1.135 4.212	1.315 3.257	0.905 20.82

V. Conclusion

It was demonstrated that the bounds based on the unilateral figure of merit are, in general, optimistic. Alternative figures of merit were proposed. Although these measures provide upper and lower bounds on the gain ratio $G_T/G_{T,u}$, the corresponding bounds are usually pessimistic.

A useful analytical approach for determining the exact bounds on the gain ratio when only the load or source impedance is varied to obtain a specified unilateral power gain was outlined in [4]. A more general procedure which can also be used to determine the bounds when the load and/or source is mismatched by an arbitrary amount or to obtain a specified G_1 and/or G_2 was outlined.

The exact bounds for arbitrary load and/or source impedances and $g_1 = 0.7943$ and/or $g_2 = 0.7943$ were determined for several microwave transistors by using a computer program based on the procedures outlined. It appears that, from the viewpoint of transducer power gain, few microwave transistors can be considered to be unilateral.

References

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